

Does a sheared flow stabilize inversely stratified fluid?

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We examine the stability of sheared flows in an inversely stratified fluid (where the density increases upward). As demonstrated by Kuo (1963), if the shear and Väsälä frequency are both constant (i.e., if the velocity and density profiles are both linear), the shear *suppresses* the Rayleigh–Taylor instability that would affect the fluid in the absence of the flow. Our main goal is to reexamine this problem for a wider class of velocity and density profiles. Using the standard linear normal-mode analysis, we consider two types of flows: jets (for which asymptotic solutions were found), and currents with a monotonic profile (which were examined numerically). It turns out that virtually any deviation from the linear profiles examined by Kuo (1963) triggers off instability. This instability, however, is restricted either spectrally or spatially, which makes it different from the usual Rayleigh–Taylor instability (in the absence of the flow, inversely stratified fluids are unstable at all points and all wavelengths). The conclusions of the paper are verified by simulation of the governing (nonlinear) equations. © 2002 American Institute of Physics.
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I. INTRODUCTION

It is well known that inversely stratified fluids (where the density increases upward) are, generally, unstable due to the Rayleigh–Taylor instability. It is also well known that sheared flows in fluids are a potential source of instability. Hence, it is logical to assume that a combination of the two effects should cause even stronger instability. Yet Kuo¹ demonstrated that shear may, in fact, *suppress* the Rayleigh–Taylor instability.

This unexpected result, derived for the case of an inversely stratified atmosphere, was later generalized for plasma governed by similar equations.^{2,3} Furthermore, the theoretical results obtained for plasma have been supported by observations of the stabilizing effect of self-generated or imposed shear flows often invoked in plasmas.⁴ Overall, in magnetically confined plasmas, the suppression of turbulence by shear seems to be so widespread that it is now believed to be universal.

It should be noted, however, that Kuo¹ and Hassam^{2,3} considered exactly the same particular case, where the shear and Väsälä frequency were both constant (i.e., the velocity and density profiles both linear). In a sense, this setting is not generic, as it is *spatially homogeneous*—which, generally, prohibits the existence of localized modes (those are normally captured by inhomogeneities of the medium). In other words, it may be possible that the conclusion about the stabilization of inverse stratification by shear is a result of a “quirk” of the model chosen, rather than a “physical effect.”

We have therefore found it worthwhile to reexamine the problem for different (nonhomogeneous) stratification/flow profiles. Two approaches were pursued: we have examined the stability of normal modes (for both fluid and plasma,

these are described by the classical Taylor–Goldstein equation^{5,6}), and also simulated the nonlinear equations governing the evolution of stable or unstable disturbances.

The structure of this paper is as follows. In Sec. II, we shall formulate the problem, and in Secs. III and IV, examine it asymptotically in the short-wave limit. It turns out, however, that the method we use is applicable only to flows where the velocity profile has an extremum (those will be referred to as jets). Thus, the important case of flows with monotonic profiles will have to be examined by direct numeric integration of the Taylor–Goldstein equation (Sec. V). In Sec. VI we shall simulate the nonlinear equations governing inversely stratified fluid, and Sec. VII contains concluding remarks.

II. FORMULATION OF THE PROBLEM

Consider a fluid of variable density $\rho(x,y,z,t)$, where x,y,z are the spatial coordinates (the z axis is directed upward), and t is the time. In terms of ρ , the pressure p , and the velocity \mathbf{u} of the flow, the governing equations can be written using the Boussinesq approximation (e.g., Ref. 7) as follows:

$$\begin{aligned} \mathbf{u}_t + (\mathbf{u} \cdot \nabla) \mathbf{u} + \frac{1}{\rho_0} \nabla p &= -\frac{\rho}{\rho_0} g \mathbf{z}, & \nabla \cdot \mathbf{u} &= 0, \\ \rho_t + \mathbf{u} \cdot \nabla \rho &= 0, \end{aligned} \quad (1)$$

where ρ_0 is the mean density, g is the acceleration due to gravity, and \mathbf{z} is the unit vector directed along the z axis (the mean density of the fluid is assumed to be equal to unity). Now, consider a parallel flow, such that the streamlines and surfaces of equal density are horizontal (see Fig. 1), perturbed by a small disturbance:

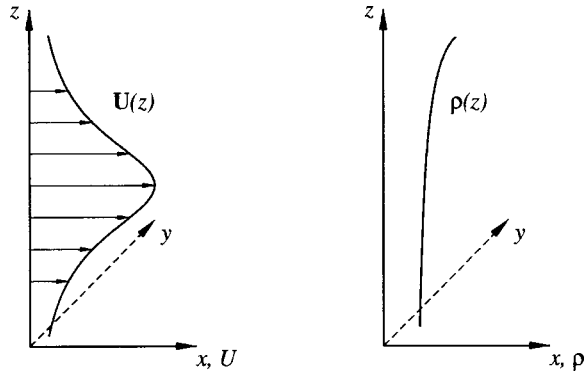


FIG. 1. Formulation of the problem: a flow in a density stratified fluid (U is the velocity, ρ is the density).

$$\mathbf{u} = \begin{bmatrix} U(z) + u'(x, y, z, t) \\ v'(x, y, z, t) \\ w'(x, y, z, t) \end{bmatrix}, \quad (2)$$

$$\rho = \bar{\rho}(z) + \rho'(x, y, z, t), \quad p = -g \int \bar{\rho}(z) dz + p'(x, y, z, t), \quad (3)$$

where $U(z)$ and $\bar{\rho}(z)$ describe the mean flow, and the dashed variables describe the disturbance. Substituting (2)–(3) into (1) and omitting nonlinear terms, we obtain

$$\begin{aligned} u'_t + Uu'_x + w'U_z + \frac{1}{\rho_0}p'_x &= 0, \\ v'_t + Uv'_x + \frac{1}{\rho_0}p'_y &= 0, \\ w'_t + Uw'_x + \frac{1}{\rho_0}p'_z &= -g \frac{\rho'}{\rho_0}, \\ u'_x + v'_y + w'_z &= 0, \quad \rho'_t + U\rho'_x + w'\bar{\rho}_z = 0. \end{aligned} \quad (4)$$

We shall assume for simplicity that the disturbance depends on x exponentially and does not depend on y at all:

$$\begin{aligned} u' &= \hat{u}(z, t)e^{ikx}, & v' &= \hat{v}(z, t)e^{ikx}, & w' &= \hat{w}(z, t)e^{ikx}, \\ p' &= \hat{p}(z, t)e^{ikx}, & \rho' &= \hat{\rho}(z, t)e^{ikx}. \end{aligned}$$

Then, eliminating \hat{u} , \hat{v} , \hat{p} , and $\hat{\rho}$, one can reduce (4)–(5) to a single equation for \hat{w} ,

$$\left(\frac{\partial}{\partial t} + ikU \right)^2 (\hat{w}_{zz} - k^2 \hat{w}) - U_{zz} \left(\frac{\partial}{\partial t} + ikU \right) ik \hat{w} - N^2 k^2 \hat{w} = 0, \quad (6)$$

where

$$N^2 = -g \frac{\bar{\rho}_z}{\rho_0}$$

is the Väsälä frequency. We are concerned with the case of inverse (statically unstable) stratification, where $\bar{\rho}$ grows with z ; hence, $N^2 < 0$. Without a loss of generality, we can put $\text{Im } N > 0$.

Next, we assume that the disturbance is *harmonic* in t , i.e.,

$$\hat{w}(z, t) = \phi(z)e^{-i\omega t}, \quad (7)$$

where $\phi(z)$ is the “normal mode,” and ω is the frequency. Upon substitution of (7) into (6), the latter turns into the Taylor–Goldstein equation (see Refs. 5, 6):

$$\phi_{zz} - \left(k^2 + \frac{kU_{zz}}{kU - \omega} - \frac{k^2 N^2}{(kU - \omega)^2} \right) \phi = 0. \quad (8)$$

Strictly speaking, (8) should be supplemented by the no-flow boundary conditions at the surface and the bottom of the ocean. We shall assume, however, that the disturbance is localized in the main thermocline (the layer of fast-changing density), which is located far away from both boundaries. In this case, we can put

$$\phi \rightarrow 0, \quad \text{as } z \rightarrow \pm \infty. \quad (9)$$

Here (8)–(9) form an eigenvalue problem for $\phi(z)$ and ω . If a disturbance exists with $\text{Im } \omega > 0$, the flow is unstable. It can be demonstrated⁸ that a sufficient condition of stability is

$$\frac{N^2}{(U_z)^2} > \frac{1}{4},$$

according to which inversely stratified fluids ($N^2 < 0$) are all potentially unstable regardless of the velocity profile. It is unclear though if this possible instability takes place in reality.

It turns out that a further analysis strongly depends on whether the velocity profile $U(z)$ has an extremum. In the next section, we shall examine the case of jets (i.e., flows, for which there exists a single point with $U_z = 0$). Physically, this corresponds to the case when the direction of the wind (and hence that of the wind-induced current) is opposite to the direction of the deep flow.

III. JETS

A. Short disturbances

Rewrite Eq. (8) in the form

$$\phi_{zz} - Q\phi = 0, \quad (10)$$

where

$$Q(z, \omega) = k^2 + \frac{kU_{zz}}{kU - \omega} - \frac{k^2 N^2}{(kU - \omega)^2},$$

and assume that the horizontal wavelength of the disturbance is *short*,

$$k \rightarrow \infty.$$

In this case, one would immediately think of the WKB asymptotic method, but, unfortunately, it would not work for the problem at hand. Indeed, the WKB method assumes $\phi(z)$ to rapidly oscillate with a wavelength of the order of $(-Q)^{1/2}$, which implies $Q < 0$. In the present case, however, $Q = k^2 + \mathcal{O}(1)$ is positive everywhere, except for the narrow region where the denominator of the second/third term is small (if such a region exists).

Instead, we shall employ the Simmons–Killworth asymptotic method,^{9,10} according to which the eigenfunction of (10) is localized spatially near a certain point z_0 . In still

water, that would be the extremum of the Väsälä frequency $N(z)$, but in the presence of a flow, the location of z_0 is unclear. It is also convenient to introduce an (undetermined, at this stage) leading-order frequency ω_0 and expand Eq. (10) about z_0 and ω_0 ,

$$\phi_{zz} - [Q(z_0, \omega_0) + Q_z(z_0, \omega_0)(z - z_0) + Q_\omega(z_0, \omega_0)(\omega - \omega_0) + \frac{1}{2}Q_{zz}(z_0, \omega_0)(z - z_0)^2 + \dots] \phi = 0. \quad (11)$$

If we truncate (11) at the zeroth order,

$$\phi_{zz} - Q(z_0, \omega_0) \phi = 0,$$

the resulting equation has constant coefficients and can be readily solved; clearly, it does not have bounded solutions for any value of ω_0 . Hence, we have to require

$$Q(z_0, \omega_0) = 0, \quad (12)$$

and examine the next order of (11),

$$\phi_{zz} - [Q_z(z_0, \omega_0)(z - z_0) + Q_\omega(z_0, \omega_0)(\omega - \omega_0)] \phi = 0.$$

This equation belongs to the Airy type and, again, does not admit localized (trapped) solutions. Indeed, those may exist only if the “potential” (the expression in the square brackets) bounds the “wave” (eigenfunction) on *both* sides, i.e., grows as $z \rightarrow \pm\infty$. Hence, we require the linear in $(z - z_0)$ term also to vanish:

$$Q_z(z_0, \omega_0) = 0, \quad (13)$$

so that the leading-order term in the expansion of Q be quadratic:

$$\phi_{zz} - [Q_\omega(z_0, \omega_0)(\omega - \omega_0) + \frac{1}{2}Q_{zz}(z_0, \omega_0)(z - z_0)^2] \phi = 0 \quad (14)$$

[it is implied here that $(\omega - \omega_0) \sim (z - z_0)^2$]. Here (14) provides a leading-order approximation for the solution of the original equation (10).

Note, however, that before solving (14) for ϕ , we need to find z_0 and ω_0 from (12) and (13). Expanding the former in powers of k^{-1} , we obtain two solutions for ω_0 :

$$\omega_0 = kU(z_0) \pm N(z_0) + O(k^{-1}), \quad (15)$$

where $+$ ($-$) corresponds to a growing (decaying) mode. Next, expanding (13), we obtain an equation for z_0 ,

$$U_z(z_0) = O(k^{-1}).$$

One can see that, to the leading order, z_0 is located at an extremum of the velocity profile, and if $U(z)$ does not have one, our asymptotic analysis yields no solutions. We conclude that, in this case, the problem does not have short-wave ($k \rightarrow \infty$) eigenfrequencies (which will be confirmed numerically in Sec. IV).

It should be noted, however, that although we seem to have found an unstable (growing) disturbance, its legitimacy is to be validated by the existence of localized solutions to (14) (the localization requirement was a precondition of all our results). In order to do this, rewrite (14) in the form

$$\phi_{\eta\eta} - (T\omega_1 + R\eta^2) \phi = 0, \quad (16)$$

where

$$\eta = z - z_0, \quad \omega_1 = \omega - \omega_0,$$

$$T = Q_\omega(z_0, \omega_0), \quad R = \frac{1}{2}Q_{zz}(z_0, \omega_0).$$

Recalling the definition of $Q(z, \omega)$, we obtain, in the short-wave limit,

$$T = \pm k^2 \left[\frac{2}{N(z_0)} + \mathcal{O}(k^{-1}) \right], \quad (17)$$

$$R = \mp k^3 \left[\frac{U_{zz}(z_0)}{N(z_0)} + \mathcal{O}(k^{-1}) \right],$$

where the upper (lower) sign corresponds to the unstable (stable) ω_0 defined by (15). We are seeking solutions that are localized at finite η (i.e., trapped near $z = z_0$),

$$\phi \rightarrow 0, \quad \text{as } \eta \rightarrow \pm\infty$$

[which, of course, agrees with the general boundary condition (9)]. Here (16) is the equation of the parabolic cylinder and has bounded solutions only if (see Ref. 11)

$$\omega_1 = - \frac{2R^{1/2}(m + \frac{1}{2})}{T}, \quad (18)$$

where $m \geq 0$ is an integer (the mode number) and $R^{1/2}$ is chosen such that

$$\text{Re } R^{1/2} > 0. \quad (19)$$

The corresponding eigenfunction is

$$\phi = \exp\left(-\frac{1}{2}R^{1/2}\eta^2\right) H_m(R^{1/4}\eta), \quad (20)$$

where H_m is the Hermite polynomial of order m .

Summarizing (15), (18), and (17), we obtain the following expression for the growth rate:

$$\text{Im } \omega \approx |N(z_0)| \left[1 - \sqrt{\left| \frac{U_{zz}(z_0)N(z_0)}{2k} \right|} \left(m + \frac{1}{2} \right) \right]. \quad (21)$$

The leading-order term here is independent of m , hence, to the leading order, disturbances of all modes grow at the same rate (determined by the local Väsälä frequency). The next-order term depends on the properties of the mode; clearly, the zeroth mode ($m=0$) is the most unstable one.

The above asymptotic results have been verified by a comparison with the exact solution of the eigenvalue problem (8), (9) obtained numerically for the case

$$U(z) = \frac{U_0}{\cosh z}, \quad N(z) = \frac{iN_0}{\cosh^2 z}. \quad (22)$$

It turns out that the asymptotic formula (21) describes the first two modes relatively well, even for moderate wave numbers k (see Fig. 2). One can also see that (21) does not work very well for the higher modes, the reason of which being that those are “less localized” than the small- m modes (it is well known that the localization radius, generally, grows with the mode number).

B. Discussion

(1) Observe that, in principle, the growth of our asymptotic solution may not be due to instability, but due to

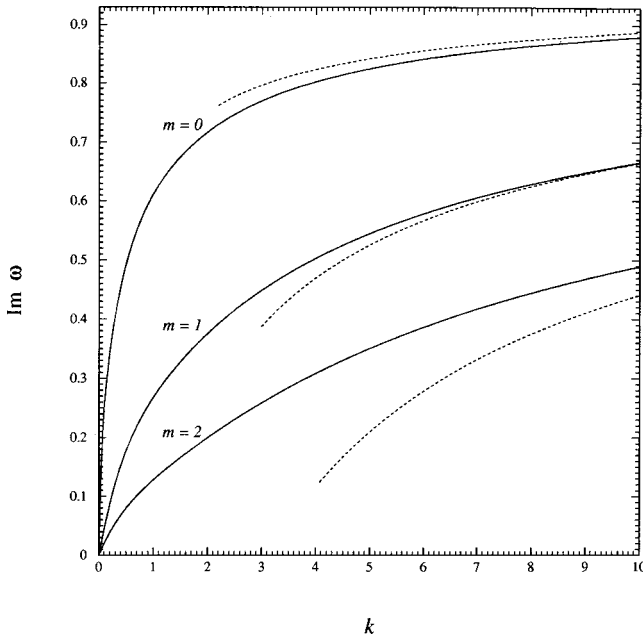


FIG. 2. Growth rate versus wave number, for flow (22) with $N_0=1$, $U_0=1$. Here m is the mode number and the dotted line shows the asymptotic growth rate [formula (21)].

the accumulation of waves coming from infinity. The simplest way to eliminate this possibility is to prove that the group velocity corresponding to the asymptotic eigenfunction (20) is directed away from the origin.

(2) To do so, consider the “original” evolution equation (6) and assume that the solution is close to, but not quite, harmonic:

$$\hat{w}(z, t) = \Phi(z, t) e^{-i\omega_0 t},$$

where Φ is a *slowly varying* function of t . Expanding (6) about $z = z_0$ in a manner that is similar to how we expanded the harmonic equation (8), we obtain

$$iT\Phi_t + R\eta^2\Phi = \Phi\eta\eta. \quad (23)$$

Next, assuming that the solution is close to a normal mode, put

$$\Phi(\eta, t) = A(\eta, t)\phi(\eta)e^{-i\omega_1 t}, \quad (24)$$

where $\phi(\eta)$ and ω_1 are the eigenfunction and eigenvalue, and $A(\eta, t)$ is a *slowly varying* function of both t and η . Substituting (24) into (23), we obtain

$$iTA_t\phi - (A\eta\eta\phi + 2A_\eta\phi\eta) = 0.$$

The propagation speed of disturbances described by this equation can be defined as the ratio of the coefficients of A_η and A_t , or rather its real part,

$$c = \text{Re} \left(-\frac{2\phi_\eta}{iT\phi} \right).$$

Using expression (20) for ϕ and taking the limit $\eta \rightarrow \infty$, we obtain

$$c \rightarrow \text{Re} \left[-\frac{2}{iT}(-R^{1/2}\eta) \right] = 2\eta \text{Im} \left(\frac{R^{1/2}}{T} \right).$$

Solution (24) does not describe waves coming from $\pm\infty$ only if c has the same sign as η , i.e.,

$$\text{Im} \left(\frac{R^{1/2}}{T} \right) \geq 0. \quad (25)$$

Substituting T and R [given by (17)] into criterion (25) and keeping in mind condition (19), one can readily verify that (25) holds for the growing mode (but does not hold for the decaying mode).

(3) It is worth mentioning that our results agree with the findings of Ref. 12, where an instability was observed near the “tip” of the jet in plasma. On the other hand, the regions with strong shear were found to be virtually stable.

(4) It is interesting to compare our results to those obtained for linearly sheared flows with constant $N^2 < 0$.¹⁻³ In the latter case, the fluid is also locally unstable, yet the flow is stable as a whole. In Ref. 2, this paradox was explained by “phase mixing” inflicted on the disturbance by the shear. In principle, this interpretation agrees with our results obtained so far, as the unstable disturbances found are localized near the “tip” of the jet, where the shear is locally zero.

An alternative explanation of the stability of linearly sheared flows with constant $N^2 < 0$ follows from the fact that those do not support localized modes (see Ref. 1), due to the invariance of the problem with respect to shifting in the z direction. Disturbances can freely travel across the flow, and the velocity shear “tears” them apart. (Observe that this argument is unapplicable to inversely stratified fluid in the absence of the flow, as, in this case, the frequency of disturbances is imaginary and their speeds are zero.) In a jet, on the other hand, disturbances are trapped. After several reflections from the “slopes” of the jet, they form a normal mode, i.e., a structure that moves as a whole *despite the sheared current*. The shear can no longer tear the unstable disturbances apart and thus prevent them from growing.

Observe that the alternative interpretation implies that instability should be expected, not only for jets, but also for flows with *any* inhomogeneities in their U and N profiles, as long as those can support (trap) a normal mode. This conclusion will be examined in the next section.

IV. FLOWS WITH MONOTONIC PROFILES

Consider a flow with monotonic $U(z)$. The short-wave method used earlier produces no eigenvalues for this case, hence, we conclude that monotonically profiled flows are unable to trap *short* disturbances. It is still unclear, however, if such flows can trap *long* disturbances. As those cannot be examined asymptotically, we shall examine them numerically, by solving the Taylor–Goldstein equation (8) with boundary condition (9).

Various monotonic profiles were tested in a wide parameter range, and in *all* cases unstable modes were found. We shall present here the results for two particular cases.

Figure 3 shows the growth rate of the zeroth (most unstable) mode for

$$U(z) = U_0 \tanh z, \quad N(z) = \frac{iN_0}{\cosh^2 z} \quad (26)$$

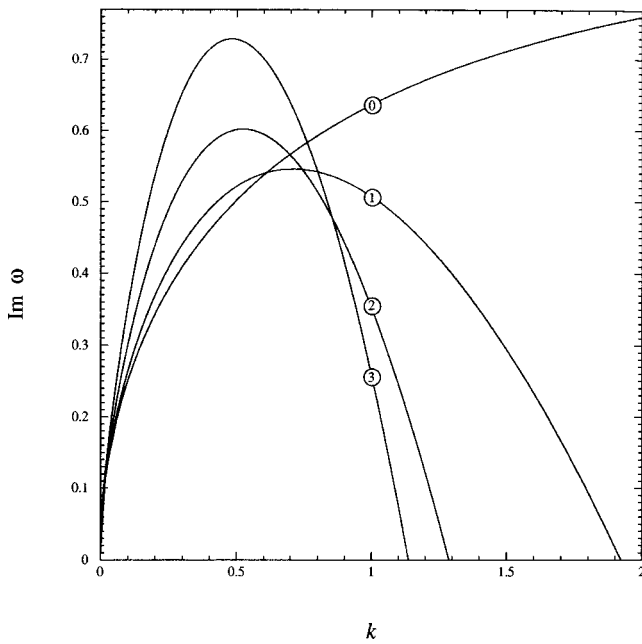


FIG. 3. The growth rate of the zeroth (most unstable) mode versus wave number, for flow (26) ($N_0=1$, $U_0=0, 1, 2, 3$). The curve number corresponds to the value of U_0 .

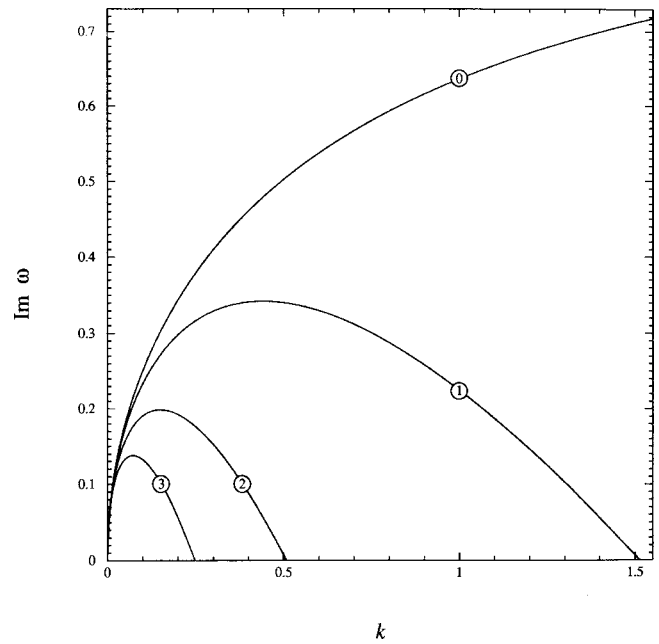


FIG. 4. The growth rate of the zeroth (most unstable) mode versus wave number, for flow (27) ($N_0=1$, $U_0=0, 1, 2, 3$). The curve number corresponds to the value of U_0 .

[observe that these profiles correspond to a constant Richardson number, $Ri=N^2/(U_z)^2$]. One can see that the shear reduces the spectral range of instability, but, at the same time, increases the growth rate. Note that, in all cases, the unstable mode disappears as $k \rightarrow \infty$, which agrees with our asymptotic results.

In order to find out if inhomogeneous density stratification can trap disturbances by itself, without the “help” of inhomogeneous shear, we have also considered

$$U(z) = U_0 z, \quad N(z) = \frac{iN_0}{\cosh^2 z}. \tag{27}$$

In this case (see Fig. 4), shear reduces both the spectral range and growth rate, but never eliminates the instability completely. We conclude that inhomogeneous stratification can alone cause instability, even in the case of constant shear.

V. SIMULATION OF THE NONLINEAR GOVERNING EQUATIONS

Note that all our conclusions have been so far obtained on the basis of normal-mode analysis. It should be kept in mind, however, that the set of eigenfunctions of the normal-mode problem (8)–(9) may be incomplete. Thus, in the cases where no unstable modes have been found, the instability can still occur with respect to disturbances of continuous spectrum. Such cases are rare, as normal modes are usually excellent “indicators” of instability, yet it is a good idea to verify our normal-mode analysis by direct numerical integration of the exact (nonlinear) equations (1).

In order to simplify the problem, we employed the two-dimensional version of (1), assuming $v=0$, and that the remaining variables are independent of y . This allows one to introduce the streamfunction ψ , such that

$$u = -\psi_z, \quad w = \psi_x,$$

and the nondimensional density

$$\tilde{\rho} = \frac{\rho}{\rho_0}.$$

Then, the governing equations (6) can be rewritten in the form (overlides omitted)

$$\nabla^2 \psi_t + J(\psi, \nabla^2 \psi) + g \rho_x = 0, \quad \rho_t + J(\psi, \rho) = 0, \tag{28}$$

where $J(\psi, \rho) = \psi_x \rho_z - \psi_z \rho_x$ is the Jacobian operator.

We tested the stability of flow (26) with $N_0=1$ and various U_0 . Equations (28) were solved in a domain, which was periodic in x with a period L_x , and bounded by walls at $z = \pm \frac{1}{2}L_z$, where we employed the no-flow boundary conditions,

$$\psi = \pm \frac{1}{2}A, \quad \text{at } z = \pm \frac{1}{2}L_z,$$

where

$$A = - \int_{-(1/2)L_z}^{(1/2)L_z} U(z) dz$$

is the net flux of the flow. Here L_z was chosen to be much larger than the “width” of $U(z)$ (it turned out that $L_z=4$ was quite sufficient). The simulations were initialized by random noise in ψ and ρ .

The results of our simulations are summarized in Table I and Fig. 5 shows examples of the evolution of $\rho(x, z, t)$ for $U_0=0$ and $U_0=1$. In the former case [Fig. 5(a)] disturbances grow and the evolution is dominated by strong convective cells. In the latter case, these cells are not allowed to grow because of the shear flow, and elongated structures are observed [Fig. 5(b)]. The flow is still (weakly) unstable, but the unstable fluctuations are much smaller than those for

TABLE I. A comparison of the linear stability results with the simulation results ($L_z=4$).

N_0	U_0	L_x	Linear stability analysis	Nonlinear simulation
1	0	4	unstable	unstable
1	0.5	4	unstable	unstable
1	1	4	unstable	unstable
1	2	4	stable	stable
1	2	5	weakly unstable	stable (just)
1	2	6	unstable	unstable

$U_0=0$, and their structure is different. When stabilization is complete (for $U_0=2$), perturbations die out completely, and the equivalent of Fig. 5 for this case would be a homogeneous square.

From the results summarized in Table I, one can also observe that shear stabilizes *short* disturbances, whereas *long* disturbances can still be unstable. Indeed, when the computational domain is extended in the x direction (when L_x is increased from 4 to 6), “new” long disturbances are introduced into the system, and these disturbances happen to be unstable. Overall, Table I shows that the results of the simulations agree well with the linear stability results (shown in Fig. 3). A similar agreement is also observed for the linearly profiled flow (27).

VI. CONCLUSIONS

The main result of this paper is the conclusion that, generally, shear *cannot* stabilize inversely stratified fluids or plasmas. The “homogeneous” (linearly profiled in U , constant- N) flow considered by Kuo¹ and Hassam^{2,3} turned out to be a special case, as virtually any inhomogeneity in the U or N profiles would destabilize it. This conclusion applies to jets (where it has been proven analytically in the short-wave limit), as well as to flows with monotonic profiles (several examples of which have been examined numerically). The instability is caused by inhomogeneities-trapped disturbances, whereas the stability of the homogeneous flow is due to its inability to trap disturbances. This conclusion, in fact, is supported by the results of Kuo,¹ who found unstable modes for the homogeneous flow *in a channel*. Indeed, although this flow’s profile cannot trap disturbances by itself, those are trapped due to reflection from the walls of the channel.

It should be mentioned, however, that, for jets, unstable disturbances are localized near its tip (i.e., where $U_z=0$). For flows with monotonic velocity profiles, in turn, although the shear does not suppress instability completely, short disturbances *are* stabilized; i.e., a threshold wave number k_c exists, such that disturbances with $k>k_c$ are stable. We conclude that, although shear does not stop instability, it still restricts it either spatially or spectrally, which is an important difference from the usual Rayleigh–Taylor instability (in the absence of the flow, inversely stratified fluids are unstable at all points and all wavelengths).

Moreover, we note that, in a domain bounded in the x direction, the instability may be inhibited *completely* if the size of the domain is sufficiently small, $L_x \leq 2\pi/k_c$. Thus,

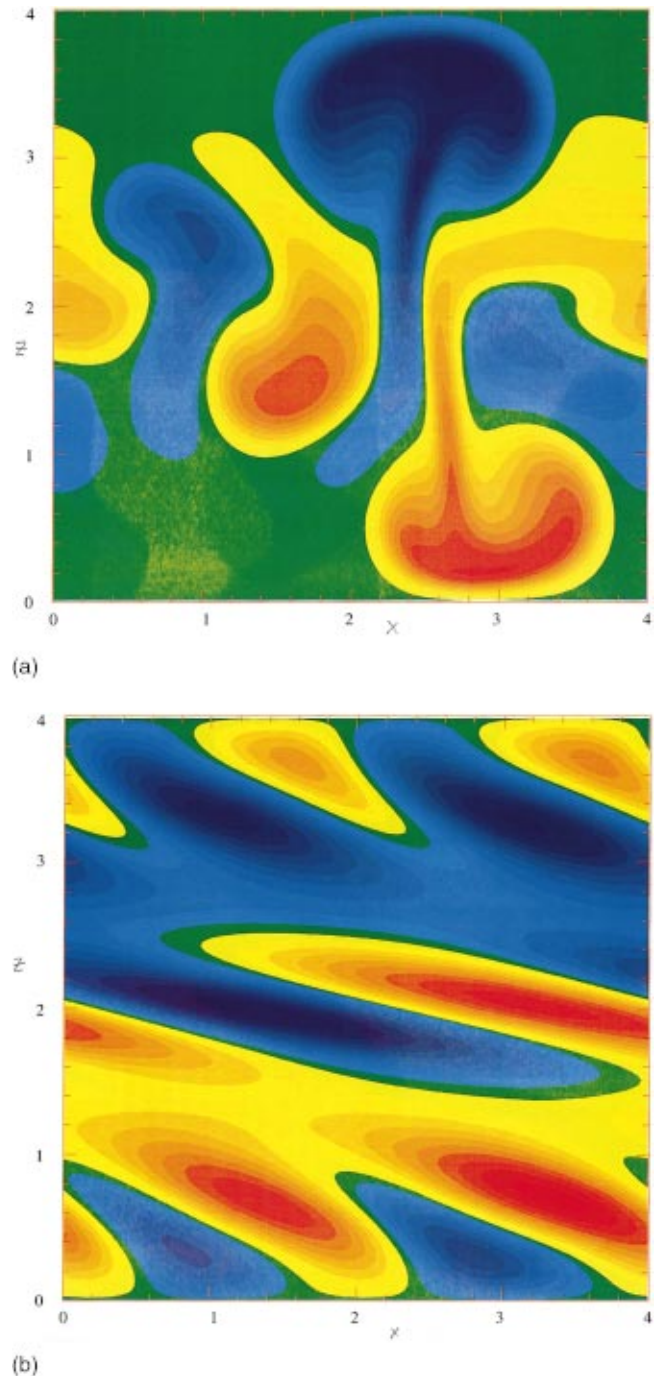


FIG. 5. (Color) Numerical solutions of Eqs. (28). (a) An unstable situation without a background flow ($U_0=0$) at $t=13$. The developing instability can be seen; large density perturbations range from -1.5 (black/blue) to 1.5 (red) in amplitude. (b) The same initial conditions as in (a), but with a shear flow ($U_0=1$). The figure shows a saturated state resulting from a slightly unstable situation at $t=200$. Density variations range from -0.03 (blue) to 0.03 (red).

for the plasma case, where the wavelengths of disturbances are limited by geometrical factors, a sufficiently strong shear may indeed stabilize the flow.

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